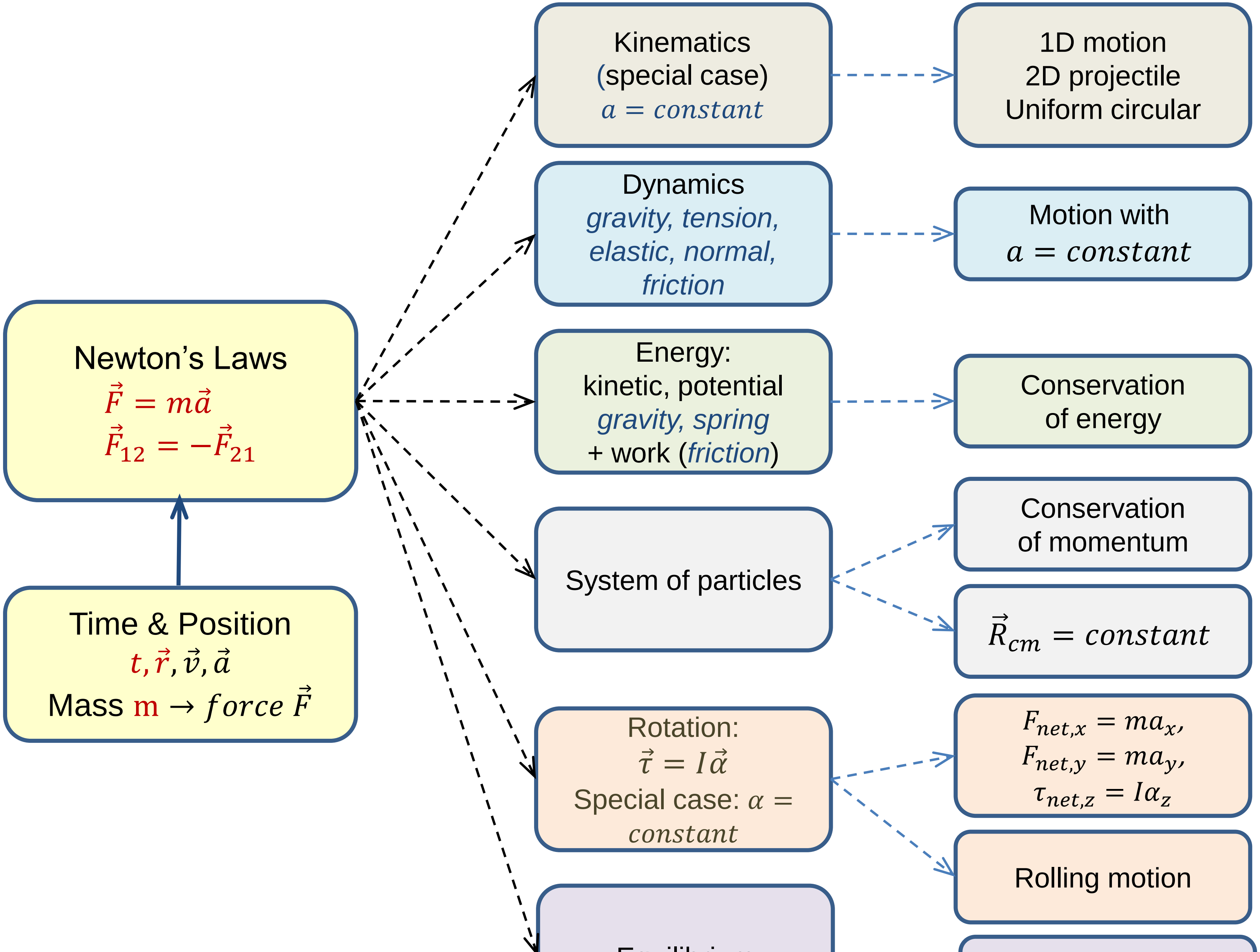


Summary: University Physics I
Physics 231 (Chapters 1-11)
Compiled by Prof. Godunov

Kinematics

1D Kinematics (definitions)

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1}, \qquad v = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{dx}{dt}$$

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}, \qquad a = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t} = \frac{d^2x}{dt^2}$$

3D Kinematics (definitions)

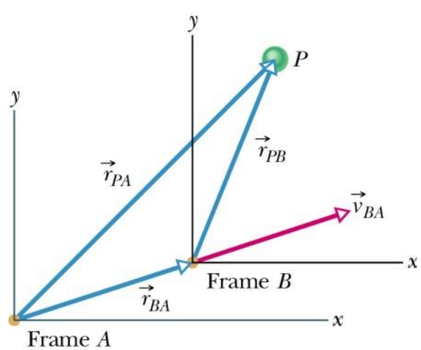
$$\vec{v}_{avg} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \quad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

$$components\!: \; v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}$$

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}, \qquad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Relative motion

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$
$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$
$$\vec{a}_{PA} = \vec{a}_{PB}$$



1D Motion $a = constant$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

If two objects – same place

$x_1 = x_2$ (same time)

2D Projectile Motion

$$x = x_0 + v_0 \cos\theta_0\, t$$
$$y = y_0 + v_0 \sin\theta_0\, t - \frac{1}{2}gt^2$$
$$v_x = v_0 \cos\theta_0$$
$$v_y = v_0 \sin\theta_0 - gt$$

Special cases:

1. $y_f = y_0 \;\; (same\ ground)$
2. $\theta_0 = 0^0 \;\; (horizontal\ launch)$
3. $\theta_0 = 90^0 \;\; (vertically\ launch)$

Uniform Circular Motion

$$a = \frac{v^2}{r}, \qquad T = \frac{2\pi}{v}$$

Dynamics

Newton's Second Law:

For any particle of mass m, the net force $\vec{F}$ on the particle is always equal to the mass m times the particle's acceleration:

$$\vec{F} = m\vec{a}$$

Newton's Third Law:

If object 1 exerts a force $\vec{F}_{21}$ on object 2, then object 2 always exerts force $\vec{F}_{12}$ on object 1 given by

$$\vec{F}_{12} = -\vec{F}_{21}$$

Forces

Gravity	$F_g = mg$	toward the ground
Normal	$N = mg\cos\theta \pm F_{external,\perp}$	perpendicular to the surface (and away)
Tension	T	along the string
Static friction	$f_s \leq \mu_s N$	parallel to surface & opposite the external force
Kinetic friction	$f_k = \mu_k N$	always opposite to direction of velocity

2D Linear (Translational) Motion

$$F_{net,x} = ma_x$$
$$F_{net,y} = ma_y$$

Centripetal force

$$F_{net,R} = \frac{mv^2}{R} \text{ (toward the center of rotation)}$$

Problem Solving: For every object we have to draw a free-body diagram.
1) Include ALL forces acting on the body matter. 2) If a problem includes more than one body - draw a separate free-body diagram for each body. 3) Not to include: any forces that the body exerts on any other body.

Energy

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

Potential Gravitational

$$U_g(y) = mgy$$

Potential Elastic

$$U_s(x) = \frac{1}{2}kx^2$$

Work

$$W(i \to f) = F_x(x_f - x_i) + F_y(y_f - y_i)$$

$$W(i \to f) = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

Power

$$P_{avg} = \frac{W}{\Delta t}, \qquad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Principle of Conservation of Energy

If all of the n forces $\vec{F}_i$ $(i = 1, \ldots n)$ acting on a particle are conservative, each with its corresponding potential energy $U_i(\vec{r})$, the total mechanical energy is constant in time

$$K_i + U_{i1} + U_{i2} + \cdots + U_{in} = K_f + U_{f1} + U_{f2} + \cdots + U_{fn}$$

Conservation of Energy with nonconservative forces (specifically with friction)

$$K_i + U_{i1} + U_{i2} + \cdots + U_{in} = K_f + U_{f1} + U_{f2} + \cdots + U_{fn} + f_k d$$

Systems of Particles

Newton's Second law for a systems of particles

$$\frac{d\vec{P}}{dt} = \vec{F}^{ext} \quad \text{or} \quad Ma_{CM} = \vec{F}^{ext}$$

Principle of Conservation of Linear Momentum: If the net force external force $\vec{F}^{ext}$ on an N-particle system is zero, the system's total mechanical momentum is constant

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_N\vec{v}_N$$

If the net force $\vec{F}^{ext}$ acting on a system is zero, then a system moves with constant velocity $\vec{v}_{CM} = contant$

And if initially $\vec{v}_{CM} = 0$, then the position of the center of mass $\vec{R}_{CM}$ does not change despite individual positions of particles may change.

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + \cdots + m_n}$$

Collisions:

Elastic: $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$ (momentum), $E_{1i} + E_{2i} = E_{1f} + E_{2f}$ (energy conserved)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

Inelastic: $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$ (momentum), $E_{1i} + E_{2i} \neq E_{1f} + E_{2f}$ (energy - NOT conserved)

Completely inelastic collision: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$

Rotation

Rotational kinetic energy

$K = \frac{1}{2} I\omega^2$

Kinetic energy of rolling

$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2 \qquad v_{cm} = \omega R$

Rotational Inertia (for a single particle)

$I = mr^2$

Rotational Inertia for common objects

$I_{solid\ cylinder} = \frac{1}{2} MR^2, I_{solid\ sphere} = \frac{2}{5} MR^2$

Translational and Rotational Dynamics (Newton's Second law)

$\vec{F}_{net} = m\vec{a}$ translational motion

$\vec{\tau} = I\vec{\alpha}$ rotational motion ($\alpha = a/r$)

for rotation in (x, y) plane

$F_{net,x} = ma_x, \quad F_{net,y} = ma_y, \quad \tau_{net,z} = I\alpha_z$

Angular variables: θ, ω, α

Relating linear and angular variables

$s = \theta r, v = \omega r, a = \alpha r$

Rotational kinematics with $\alpha = const$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

Torque

$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\varphi$

$\tau_z = xF_y - yF_x$ for rotation in (x, y) plane

Angular Momentum

$\vec{L} = \vec{r} \times \vec{p}$ (a particle), $\vec{L} = I\vec{\omega}$ (rigid body)

$\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$ Newton's second law

if $\vec{\tau}_{net} = 0$ then $\vec{L} = constant$ or $I_i\omega_i = I_f\omega_f$

Equilibrium (translational + rotational)

$F_{net,x} = 0$ balance of forces

$F_{net,y} = 0$ balance of forces

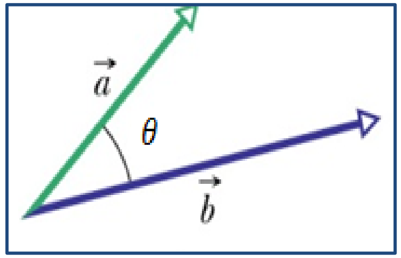
$\tau_{net,z} = 0$ balance of torques

Math and more

Scalar (dot) product

$$c = \vec{a} \cdot \vec{b} = ab\cos\theta$$

$$c = \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

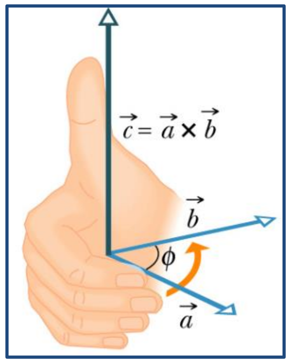


Vector (cross) product

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin\phi$$

For vectors in (xy) plane

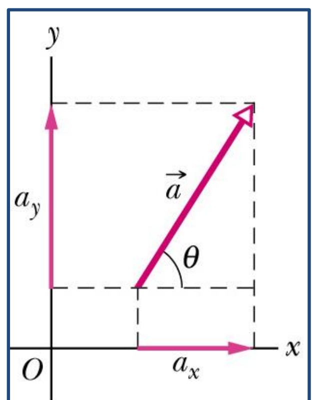
$$c_z = a_x b_y - a_y b_x$$



Vectors (components)

$$a_x = a\cos\theta,\ a_y = a\sin\theta$$

$$a = \sqrt{a_x^2 + a_y^2}, \qquad \theta = \arctan\frac{a_y}{a_x}$$



Quadratic equation

$$ax^2 + bx + c = 0,$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

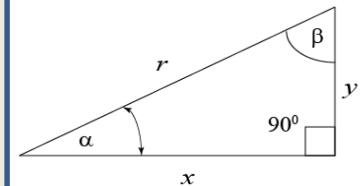
Trigonometry

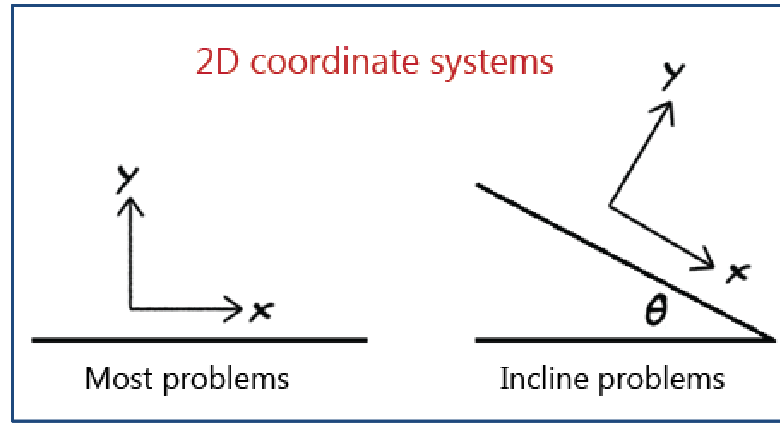
$$x^2 + y^2 = r^2$$

$$\alpha + \beta = 90^0$$

$$x = r\cos\alpha$$

$$y = r\sin\alpha$$





Unit conversion (example)

$$130\ km/h = \left(\frac{130\ km}{1\ h}\right)\left(\frac{1000\ m}{1\ km}\right)\left(\frac{1\ h}{3600\ s}\right)$$

$$= 36.1\ m/s$$